Period 1, Sept 6, 2024

2. Find the following limits without a calculator. Show your work.

a) $\lim_{x\to-\infty} \frac{\sqrt{12x^2-4}}{x+2} = \frac{\sqrt{12x^2}}{x} = \frac{\sqrt{x^2}\cdot\sqrt{12}}{x}$

$\frac{|x|\cdot\sqrt{12}}{x} = \frac{+RBN\cdot\sqrt{12}}{-RBN}$

$-\sqrt{12}$

b) $\lim_{x\to-\infty} \frac{5x^3-2x}{7+2x^4}$

$\lim_{x\to-\infty} \frac{5x^3}{2x^4} = \lim_{x\to-\infty} \frac{5}{2x} = \frac{5}{2(-RBN)} = 0$

c) $\lim_{x\to\infty} \frac{3e^x}{500x^{400}} = \lim_{x\to\infty} \frac{3\cdot RBN}{500\cdot RBN}$

$\lim_{x\to\infty} e^x > x^{400}$

∞

d) $\lim_{x\to\infty} e^{-x}\cos x$

Smallest

Biggest

$\lim_{x\to\infty} \frac{\cos x}{e^x} = \frac{-1}{\infty}$ or $\frac{1}{\infty} = 0$

e) $\lim_{x\to\infty} \left(\frac{3}{x}+2\right)\left(\frac{4x^2-1}{x^2}\right)$

$(0+2)\left(\frac{4x^2}{x^2}\right)$

$(2)(4) = 8$

$\frac{x}{x}\cdot\left(\frac{3}{x}+2\right) = \frac{3x}{x^2} + \frac{2x}{x}$

$\frac{3}{x} + \frac{2x}{x} = \frac{3+2x}{x}$

5. Sketch a function that satisfies the stated conditions. Include any asymptotes.

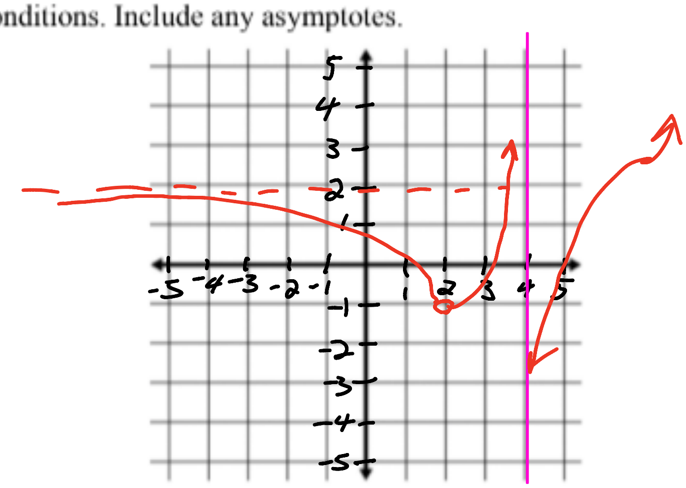
$\lim_{x\to2} f(x) = -1$

$\lim_{x\to4^-} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = 2$

$\lim_{x\to4^+} f(x) = -\infty$

$\lim_{x\to\infty} f(x) = \infty$



4. Find all the horizontal asymptotes of the function $y = \frac{8+2^x}{2+2^x}$

$\lim_{x\to\infty} \frac{8+2^x}{2+2^x} = \lim_{x\to\infty} \frac{2^x}{2^x} = 1$

$2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

$2^{-11} = \frac{1}{2^{11}} = \frac{1}{2048}$

$\lim_{x\to-\infty} \frac{8+2^x}{2+2^x} = \frac{8+0}{2+0} = \frac{8}{2} = 4$

$2^{-12} = \frac{1}{2^{12}} = \frac{1}{4096}$

Show work.

a) $f(x) = \frac{x-2}{2x^2+3x-5}$

$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} \frac{x}{2x^2} = \lim_{x\to\infty} \frac{1}{2x} = 0 = \frac{1}{RBN}$

$\lim_{x\to-\infty} F(x) = \lim_{x\to-\infty} \frac{x}{2x^2} = \lim_{x\to-\infty} \frac{1}{2x} = 0$

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e) $f(x) = \frac{\sqrt{x^2+2}}{x-5} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{x} = \lim_{x\to\infty} \frac{|x|}{x} = 1$

$\lim_{x\to-\infty} \frac{\sqrt{x^2}}{x} = \frac{|x|}{x} = -1$

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b) $f(x) = \frac{4x^3-2x+1}{x^2-2x+1}$

$\lim_{x\to\infty} \frac{4x^3}{x^2} = 4x = \infty = DNE$

$\lim_{x\to-\infty} \frac{4x^3}{x^2} = 4x = -\infty = DNE$

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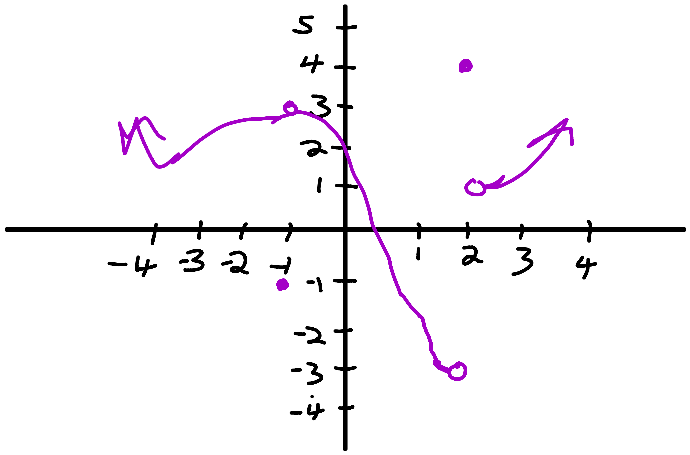
d) $f(x) = \frac{|x|}{x}$

$\lim_{x\to\infty} F(x) = 1$

$\lim_{x\to\infty} -F(x) = -1$

$\frac{|2{,}000{,}000{,}000|}{2{,}000{,}000{,}000} = \frac{2{,}000{,}000{,}000}{2{,}000{,}000{,}000} = 1$

$\frac{|-2{,}000{,}000{,}000|}{-2{,}000{,}000{,}000} = \frac{2{,}000{,}000{,}000}{-2{,}000{,}000{,}000} = -1$



$F(-1) = -1$

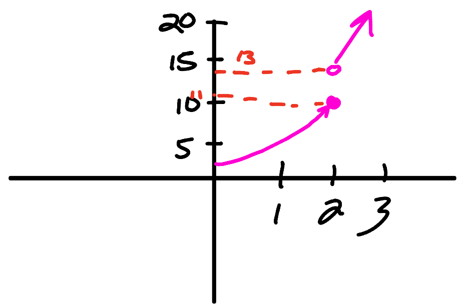
$$F(x) = \begin{cases} 2x^2+3 & x<2 \\ 11 & x=2 \\ 5x+3 & x>2 \end{cases}$$

$\lim\limits_{x\to 2^-} F(x) = \lim\limits_{x\to 2^-} 2x^2+3 = 2\cdot 2^2+3 = 11$

Not The Same

$\lim\limits_{x\to 2^+} F(x) = \lim\limits_{x\to 2^+} 5x+3 = 5\cdot 2+3 = 13$

$\lim\limits_{x\to 2} F(x) = \emptyset = DNE$



$$F(x) = \begin{cases} 7x+1 & x<2 \\ 15 & x=2 \\ x^3+2x & x>2 \end{cases}$$

$\lim\limits_{x\to 2^+} F(x) = \lim\limits_{x\to 2^+} x^3+2x = 2^3+2\cdot 2 = 12$

Not The Same

$\lim\limits_{x\to 2^-} F(x) = \lim\limits_{x\to 2^-} 7x+1 = 7\cdot 2+1 = 15$

$\lim\limits_{x\to 2} F(x) = DNE = \emptyset$

3.

$\lim_{x \to c} F(x) = -8 \qquad \lim_{x \to c} g(x) = -7$

$\lim_{x \to c} [3F(x) - g^2(x)] = 3 \cdot -8 - (-7)^2 = -24 - 49 = -73$

$\lim_{x \to c} F(x) = 9 \qquad \lim_{x \to c} g(x) = -5$

$\lim_{x \to c} [\frac{1}{3}F(x) + g(x)] = \frac{1}{3} \cdot 9 + -5 = 3 - 5 = -2$

$F(-3) = \emptyset$

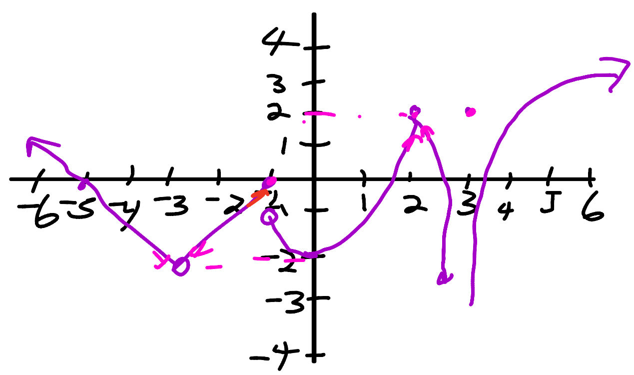
$\lim_{x \to -3} F(x) = -2$

$F(-1) = 0$

$\lim_{x \to -1} F(x) = DNE$

$\lim_{x \to -1^-} F = 0$

$\lim_{x \to -1^+} F = -1$



$F(2) = 2$

$\lim_{x \to 2} F(x) = 2$

$F(3) = 2$

$\lim_{x \to 3} F(x) = -\infty = DNE = \emptyset$

$\lim_{x \to 1} \cos \frac{\pi x}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$\lim_{x \to 5} \frac{2x-10}{x^2-25} = \lim_{x \to 5} \frac{2\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} = \frac{2}{5+5} = \frac{2}{10} = \frac{1}{5}$

$\lim_{x \to 7} \frac{2x-14}{x^2-49} = \lim_{x \to 7} \frac{2\cancel{(x-7)}}{\cancel{(x-7)}(x+7)} = \frac{2}{7+7} = \frac{2}{14} = \frac{1}{7}$

$$\lim_{x\to 2} \frac{(\sqrt{x+7}-3)}{(x-2)}\cdot\frac{(\sqrt{x+7}+3)}{(\sqrt{x+7}+3)} = \frac{x+7+\cancel{3\sqrt{x+7}}-\cancel{3\sqrt{x+7}}-9}{(x-2)(\sqrt{x+7}+3)} = \frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{x+7}+3)}$$

$$\frac{\sqrt{2+7}-3}{2-2} = \frac{\sqrt{9}-3}{0} = \frac{3-3}{0} = \frac{0}{0} = \phi$$

$$\lim_{x\to 2} \frac{1}{\sqrt{x+7}+3}$$

$$\frac{1}{\sqrt{2+7}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3}$$

$$\frac{1}{6}$$

$$\lim_{x\to 4} \frac{(\sqrt{x+12}-4)}{(x-4)}\cdot\frac{(\sqrt{x+12}+4)}{(\sqrt{x+12}+4)} = \frac{x+12+\cancel{4\sqrt{x+12}}-\cancel{4\sqrt{x+12}}-16}{(x-4)(\sqrt{x+12}+4)}$$

$$\frac{\cancel{(x-4)}}{\cancel{(x-4)}(\sqrt{x+12}+4)} = \frac{1}{\sqrt{x+12}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4}$$

$$\frac{1}{8}$$

$$\lim_{x\to 0} \frac{\sin 5x}{2x} = \lim_{x\to 0} \frac{5\cdot\sin 5x}{2\cdot 5x} = \frac{5}{2}\cdot 1 = \frac{5}{2}$$

$$\frac{5}{5}\cdot\frac{\sin 5x}{2x} = \frac{5\cdot\sin 5x}{2\cdot 5x}$$

BAD!!!!

$$\sin 5x \neq 5\sin x$$

$$\sin 5\cdot\frac{\pi}{2} \neq 5\cdot\sin\frac{\pi}{2}$$

$$\sin\frac{5\pi}{2} \neq 5\cdot 1$$

$$\sin 2\tfrac{1}{2}\pi \neq 5$$

$$1 \neq 5$$

$$\lim_{x\to 0} \frac{2}{2}\cdot\frac{\sin 2x}{5x} = \lim_{x\to 0} \frac{2\,\boxed{\sin 2x}}{5\,\boxed{2x}} = \frac{2}{5}\cdot 1$$

$$1$$

$$= \frac{2}{5}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$$

$$\frac{4\cdot 1}{4\cdot(x+4)} - \frac{1(x+4)}{4(x+4)}$$

$$\frac{4}{4(x+4)} - \frac{x+4}{4(x+4)} = \frac{\cancel{4}-x-\cancel{4}}{4(x+4)} = \frac{-x}{4(x+4)}$$

$$\lim_{x\to 0} \frac{\frac{-x}{4(x+4)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{-\cancel{x}}{4(x+4)}\cdot\frac{1}{\cancel{x}}$$

$$\frac{-1}{4(0+4)} = \frac{-1}{4\cdot 4} = -\frac{1}{16}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+5} - \frac{1}{5}}{x}$$

$$\frac{5\cdot 1}{5(x+5)} - \frac{1(x+5)}{5(x+5)}$$

$$\frac{5}{5(x+5)} - \frac{x+5}{5(x+5)} = \frac{\cancel{5}-x-\cancel{5}}{5(x+5)}$$

$$\lim_{x\to 0} \frac{\frac{-x}{5(x+5)}}{\frac{x}{1}} = \lim_{x\to 0} \frac{-\cancel{x}}{5(x+5)} \cdot \frac{1}{\cancel{x}} = \lim_{x\to 0} \frac{-1}{5(x+5)} = \frac{-1}{5(0+5)} = \frac{-1}{5\cdot 5} = \frac{-1}{25}$$

$$-x^2+4x-1 \le F(x) \le x^2-4x+7 \qquad \lim_{x\to 2} F(x)$$

$$-(2)^2+4(2)-1 \qquad (2)^2-4(2)+7$$

$$-4+8-1 \qquad 4-8+7$$

$$3 \le \lim_{x\to 3} F(x) \le 3$$

Squeeze Theorem

$$\lim_{x\to 2} -x^2+4x-1 \le \lim_{x\to 2} F(x) \le \lim_{x\to 2} x^2-4x+7 \quad \lim_{x\to 3}$$

$F(x)=3$

$3 \le$

L = constant

$$L \le F(x) \le L$$